OCR
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# Tuesday 20 June 2023 – Afternoon

## A Level Mathematics A

**H240/03** Pure Mathematics and Mechanics

**Time allowed: 2 hours**

*6888578800*

**You must have:**
- the Printed Answer Booklet
- a scientific or graphical calculator

**QP**

**INSTRUCTIONS**
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

**ADVICE**
- Read each question carefully before you start your answer.

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# Formulae
# A Level Mathematics A (H240)

## Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

## Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

## Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

## Differentiation

| $\mathrm{f}(x)$ | $\mathrm{f}'(x)$ |
|---|---|
| $\tan kx$ | $k\sec^2 kx$ |
| $\sec x$ | $\sec x \tan x$ |
| $\cot x$ | $-\mathrm{cosec}^2 x$ |
| $\mathrm{cosec}\, x$ | $-\mathrm{cosec}\, x \cot x$ |

Quotient rule $y = \dfrac{u}{v},\ \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

## Differentiation from first principles

$\mathrm{f}'(x) = \lim_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

## Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

## Small angle approximations

$\sin\theta \approx \theta,\ \cos\theta \approx 1 - \frac{1}{2}\theta^2,\ \tan\theta \approx \theta$ where $\theta$ is measured in radians

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**Trigonometric identities**

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

**Numerical methods**

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

**Probability**

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

**Standard deviation**

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

**The binomial distribution**

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of $X$ is $np$, variance of $X$ is $np(1-p)$

**Hypothesis test for the mean of a normal distribution**

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

**Percentage points of the normal distribution**

If $Z$ has a normal distribution with mean 0 and variance 1 then, for each value of $p$, the table gives the value of $z$ such that $\mathrm{P}(Z \leqslant z) = p$.

| $p$ | 0.75 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 | 0.9975 | 0.999 | 0.9995 |
|---|---|---|---|---|---|---|---|---|---|
| $z$ | 0.674 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 2.807 | 3.090 | 3.291 |

**Kinematics**

| Motion in a straight line | Motion in two dimensions |
|---|---|
| $v = u + at$ | $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ |
| $s = ut + \frac{1}{2}at^2$ | $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ |
| $s = \frac{1}{2}(u + v)t$ | $\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$ |
| $v^2 = u^2 + 2as$ | |
| $s = vt - \frac{1}{2}at^2$ | $\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$ |

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## Section A
## Pure Mathematics

**1** Using logarithms, solve the equation

$4^{2x+1} = 5^x$,

giving your answer correct to **3** significant figures. **[3]**

**2** **(a)** Express $3\sin x - 4\cos x$ in the form $R\sin(x-\alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of $\alpha$ correct to **4** significant figures. **[3]**

**(b)** Hence solve the equation $3\sin x - 4\cos x = 2$ for $0° < x < 90°$, giving your answer correct to **3** significant figures. **[2]**

**3** The cubic polynomial $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = x^3 + px + q$, where $p$ and $q$ are constants.

**(a)** **(i)** Given that $\mathrm{f}'(2) = 13$, find the value of $p$. **[2]**

**(ii)** Given also that $(x-2)$ is a factor of $\mathrm{f}(x)$, find the value of $q$. **[2]**

The curve $y = \mathrm{f}(x)$ is translated by the vector $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

**(b)** Using the values from part **(a)**, determine the equation of the curve after it has been translated. Give your answer in the form $y = x^3 + ax^2 + bx + c$, where $a$, $b$ and $c$ are integers to be found. **[4]**

**4** A circle $C$ has equation $x^2 + y^2 - 6x + 10y + k = 0$.

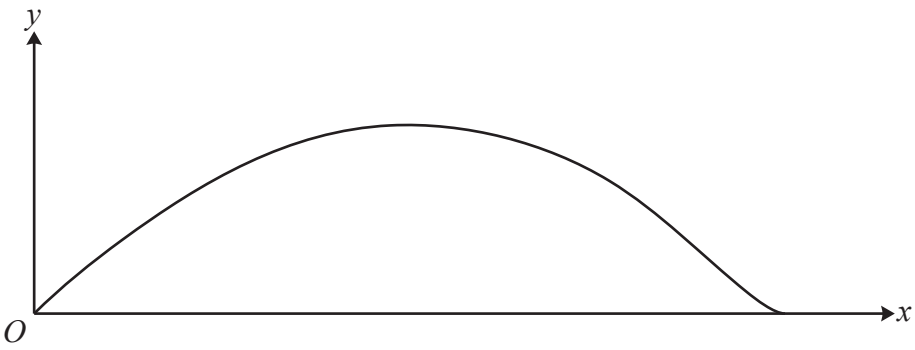
**(a)** Find the set of possible values of $k$. **[2]**

**(b)** It is given that $k = -46$.

Determine the coordinates of the **two** points on $C$ at which the gradient of the tangent is $\frac{1}{2}$. **[5]**

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**5** A mathematics department is designing a new emblem to place on the walls outside its classrooms. The design for the emblem is shown in the diagram below.




The emblem is modelled by the region between the $x$-axis and the curve with parametric equations

$$x = 1 + 0.2t - \cos t, \qquad y = k\sin^2 t,$$

where $k$ is a positive constant and $0 \leqslant t \leqslant \pi$.

Lengths are in metres and the area of the emblem must be $1\,\text{m}^2$.

**(a)** Show that $k\int_0^{\pi}(0.2 + \sin t - 0.2\cos^2 t - \sin t\cos^2 t)\,\mathrm{d}t = 1$. **[3]**

**(b)** Determine the exact value of $k$. **[6]**

**6** The first, third and fourth terms of an arithmetic progression are $u_1$, $u_3$ and $u_4$ respectively, where

$$u_1 = 2\sin\theta, \qquad u_3 = -\sqrt{3}\cos\theta, \qquad u_4 = \tfrac{7}{2}\sin\theta,$$

and $\frac{1}{2}\pi < \theta < \pi$.

**(a)** Determine the exact value of $\theta$. **[3]**

**(b)** Hence determine the value of $\sum_{r=1}^{100} u_r$. **[3]**

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**7** A car $C$ is moving horizontally in a straight line with velocity $v\,\text{m s}^{-1}$ at time $t$ seconds, where $v > 0$ and $t \geqslant 0$. The acceleration, $a\,\text{m s}^{-2}$, of $C$ is modelled by the equation

$$a = v\left(\frac{8t}{7+4t^2} - \frac{1}{2}\right).$$

**(a) In this question you must show detailed reasoning.**

Find the times when the acceleration of $C$ is zero. **[3]**

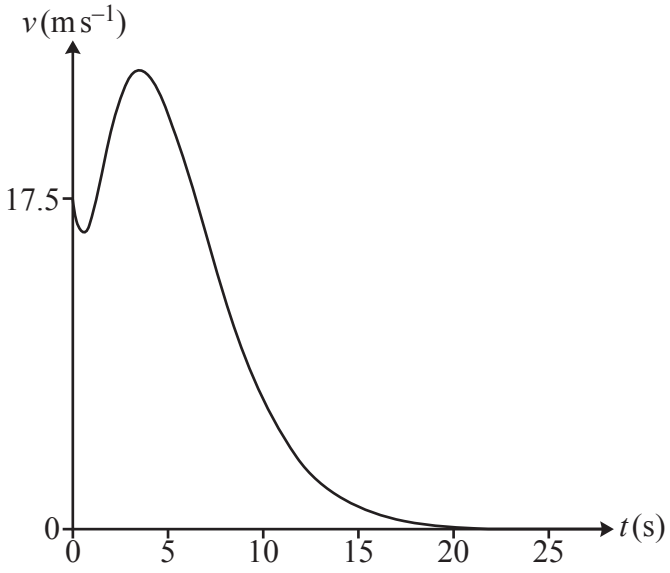
At $t = 0$ the velocity of $C$ is $17.5\,\text{m s}^{-1}$ and at $t = T$ the velocity of $C$ is $5\,\text{m s}^{-1}$.

**(b)** By setting up and solving a differential equation, show that $T$ satisfies the equation

$$T = 2\ln\left(\frac{7+4T^2}{2}\right).$$ **[6]**

**(c)** Use an iterative formula, based on the equation in part **(b)**, to find the value of $T$, giving your answer correct to **4** significant figures. Use an initial value of 11.25 and show the result of each step of the iteration process. **[2]**

**(d)** The diagram below shows the velocity-time graph for the motion of $C$.



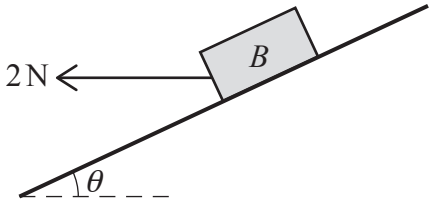

Find the time taken for $C$ to decelerate from travelling at its maximum speed until it is travelling at $5\,\text{m s}^{-1}$. **[1]**

## Section B
## Mechanics

**8** A particle $P$ moves with constant acceleration $(3\mathbf{i}-2\mathbf{j})\,\mathrm{m\,s^{-2}}$. At time $t = 4$ seconds, $P$ has velocity $6\mathbf{i}\,\mathrm{m\,s^{-1}}$.

Determine the speed of $P$ at time $t = 0$ seconds. **[4]**

**9**




A block $B$ of weight 10 N lies at rest in equilibrium on a rough plane inclined at $\theta$ to the horizontal. A horizontal force of magnitude 2 N, acting above a line of greatest slope, is applied to $B$ (see diagram).

**(a)** Complete the diagram in the Printed Answer Booklet to show all the forces acting on $B$. **[1]**

It is given that $B$ remains at rest and the coefficient of friction between $B$ and the plane is 0.8.

**(b)** Determine the greatest possible value of $\tan\theta$. **[5]**

**10** A particle $P$ of mass $m$ kg is moving on a smooth horizontal surface under the action of two constant horizontal forces $(-4\mathbf{i}+2\mathbf{j})$ N and $(a\mathbf{i}+b\mathbf{j})$ N. The resultant of these two forces is $\mathbf{R}$ N. It is given that $\mathbf{R}$ acts in a direction which is parallel to the vector $-\mathbf{i}+3\mathbf{j}$.

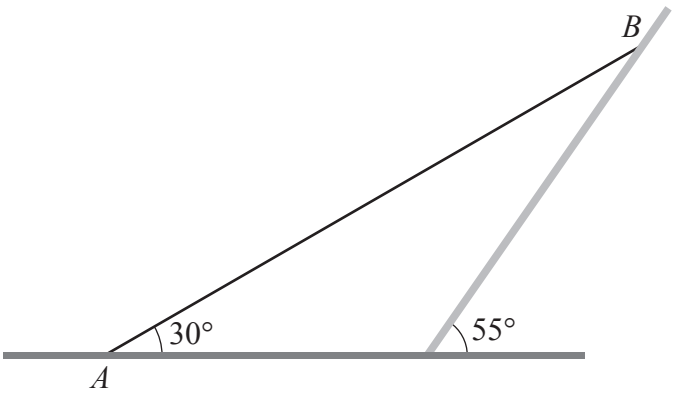
**(a)** Show that $3a+b=10$. **[3]**

It is given that $a = 6$ and that $P$ moves with an acceleration of magnitude $5\sqrt{10}\,\mathrm{m\,s^{-2}}$.

**(b)** Determine the value of $m$. **[4]**

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**11**



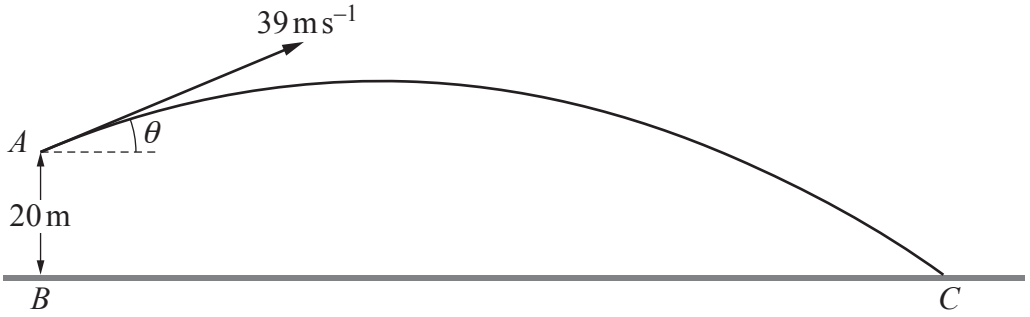

A uniform rod $AB$, of weight 20 N and length 2.8 m, rests in equilibrium with the end $A$ in contact with rough horizontal ground and the end $B$ resting against a smooth wall inclined at 55° to the horizontal. The rod, which rests in a vertical plane that is perpendicular to the wall, is inclined at 30° to the horizontal (see diagram).

**(a)** Show that the magnitude of the force acting on the rod at $B$ is 9.56 N, correct to **3** significant figures. **[3]**

**(b)** Determine the magnitude of the contact force between the rod and the ground. Give your answer correct to **3** significant figures. **[5]**

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**12 In this question you should take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.**




A small ball $P$ is projected from a point $A$ with speed $39\,\text{m}\,\text{s}^{-1}$ at an angle of elevation $\theta$, where $\sin\theta = \frac{5}{13}$ and $\cos\theta = \frac{12}{13}$. Point $A$ is 20 m vertically above a point $B$ on horizontal ground. The ball first lands at a point $C$ on the horizontal ground (see diagram).

The ball $P$ is modelled as a particle moving freely under gravity.

**(a)** Find the maximum height of $P$ above the ground during its motion. **[3]**

The time taken for $P$ to travel from $A$ to $C$ is $T$ seconds.

**(b)** Determine the value of $T$. **[3]**

**(c)** State **one** limitation of the model, other than air resistance or the wind, that could affect the answer to part **(b)**. **[1]**

At the instant that $P$ is projected, a second small ball $Q$ is released from rest at $B$ and moves towards $C$ along the horizontal ground.

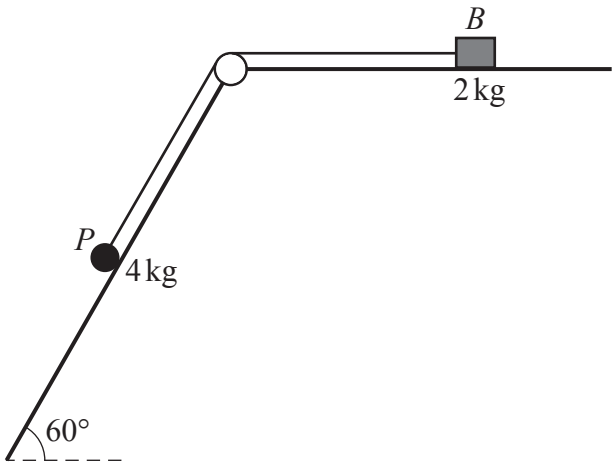
At time $t$ seconds, where $t \geqslant 0$, the velocity $v\,\text{m}\,\text{s}^{-1}$ of $Q$ is given by

$$v = kt^3 + 6t^2 + \tfrac{3}{2}t,$$

where $k$ is a positive constant.

**(d)** Given that $P$ and $Q$ collide at $C$, determine the acceleration of $Q$ immediately before this collision. **[6]**

**13**




The diagram shows a small block $B$, of mass 2 kg, and a particle $P$, of mass 4 kg, which are attached to the ends of a light inextensible string. The string is taut and passes over a small smooth pulley fixed at the intersection of a horizontal surface and an inclined plane. The particle can move on the inclined plane, which is rough, and which makes an angle of 60° with the horizontal. The block can move on the horizontal surface, which is also rough.

The system is released from rest, and in the subsequent motion $P$ moves down the plane and $B$ does not reach the pulley.

It is given that the coefficient of friction between $P$ and the inclined plane is twice the coefficient of friction between $B$ and the horizontal surface.

**(a)** Determine, in terms of $g$, the tension in the string. **[7]**

When $P$ is moving at $2\,\text{m s}^{-1}$ the string breaks. In the 0.5 seconds after the string breaks $P$ moves 1.9 m down the plane.

**(b)** Determine the deceleration of $B$ after the string breaks. Give your answer correct to **3** significant figures. **[5]**

**END OF QUESTION PAPER**

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